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AN APPROACH TO DESIGN AND OPTIMIZATION OF LOW-THRUST TRAJECTORIES WITH GRAVITY ASSISTS*

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Low-thrust trajectories using gravity-assist maneuvers can provide excellent performance on missions to a wide variety of destinations in the solar system. A two-step approach to design such trajectories is described. The first step uses an analytic, shape-based method to perform a broad search for promising cases. The second step improves the best candidates using an efficient parameter optimization method. Examples are presented for missions to Vesta, Tempel 1, Ceres, Jupiter, and Pluto.

INTRODUCTION

Before February 5, 1974, the interplanetary gravity assist was an untested idea. On that day, now a generation ago, Mariner 10 used a gravity assist at Venus to achieve the first close flyby of Mercury. Thus began a new era in which the gravity-assist maneuver became a trusted and useful technique for enabling deep-space exploration.

Another technology, solar electric propulsion (SEP), has only recently proven its worthiness for interplanetary missions. In March 2001, the NSTAR thruster on Deep Space 1 exceeded 10,000 hours of operation in interplanetary space. The specific impulse (I_{sp}) of this thruster is about ten times that of chemical rockets, enabling large savings in propellant costs.[1] The increased I_{sp} comes at the cost of low thrust, but that is offset by the ability to thrust for long durations.

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Used together, gravity-assist maneuvers and low-thrust propulsion constitute a new type of trajectory that we refer to as a low-thrust gravity-assist (LTGA) trajectory. The LTGA concept provides a means to shorten mission duration and reduce propellant requirements.

Unfortunately, designing low-thrust gravity-assist trajectories is a formidable task. Unlike ballistic trajectories, optimal low-thrust arcs are not simple geometric shapes (like conic sections). In fact, there are infinitely many possible low-thrust arcs between any two target bodies. The task is further complicated by timing (i.e. phasing) considerations, especially when the trajectory involves multiple gravity assists.

Despite these difficulties, LTGA trajectories have been found and proposed for missions ranging from Mercury orbiters to heliopause explorers.[2–23] The Deep Space 1 mission has shown that such missions are not only theoretically possible, they are also technically feasible.[1]

In this paper, we describe an approach for designing and optimizing such trajectories.

METHODOLOGY

Step 1: Broad Search

When optimizing LTGA trajectories, a good initial guess is essential for optimizer convergence. We also want the computation of initial guesses to be quick, so that a broad range of trajectories can be considered. Thus, we wish to avoid the time consuming approach of numerical propagation.

The first step in our process assumes a two-body model and patches together coast and thrust arcs. We employ conic sections for the coasting arcs and *exponential sinusoids*[12] for the thrusting arcs. Exponential sinusoids are geometric curves that can be parameterized in polar coordinates (r, θ) as

$$r = k_0 \exp(k_1 \sin(k_2\theta + \phi)) \tag{1}$$

where k_0, k_1, k_2, and ϕ are constants. Using this patched-arc model, no differential equations need to be propagated numerically. Instead, we propagate the trajectories analytically. These LTGA trajectories are constructed simply by solving transcendental equations and quadratures. This enables the rapid evaluation of a broad range of trajectories.

The mathematical and algorithmic details of this first step are worked out by Petropoulos et al.,[12] Petropoulos and Longuski,[13,15] and Petropoulos.[14] Petropoulos and Longuski implement the algorithms as an extension of the Satellite Tour Design Program (STOUR). With STOUR,[24] we design LTGA trajectories using a wide variety of design criteria. The most basic of these criteria is deciding when to apply coast and thrust arcs. STOUR provides four options for the path between any two target bodies: 1) a pure thrust arc, 2) a pure coast arc, 3) a coast/thrust arc, and 4) a thrust/coast arc. We specify the desired type of trajectory between the bodies and, in the case of the latter two, the distance from the central body (e.g. the sun) where we switch between coasting and thrusting.

Other user-specified design parameters include the launch date, launch V_∞, maximum total time of flight, maximum time of flight on each leg (e.g. Earth to Mars), maximum arrival V_∞, maximum propellant mass fraction, and a fixed engine I_{sp}. These parameters and others allow for broad searches and for more refined investigations of promising regions of the design space. Generally, we desire rendezvous missions with low arrival V_∞, low propellant mass fraction, and low time of flight for a given launch V_∞. For flyby missions, the arrival V_∞ magnitude is less important.

We develop a cost function to evaluate the trajectories found by STOUR. This cost function enables us to eliminate undesirable trajectories (e.g. with high arrival V_∞ and high propellant mass fraction). We calculate a "total propellant mass fraction" that includes the launch V_∞, the arrival V_∞ (in the case of a rendezvous), and STOUR's thrust arc propellant mass fraction (the ratio of propellant mass to initial spacecraft mass). The launch V_∞ is included in the cost function via Tsiolkovsky's rocket equation,[25,26] as if it were a ΔV. We use a chemical I_{sp} of 350 seconds (I_{sp1}) for the launch. For the arrival V_∞, we assume an optimizer would eliminate the excess velocity over the course of the low-thrust arc. We therefore employ the rocket equation with an I_{sp} of 3000 seconds (I_{sp2}) typical for highly efficient low-thrust engines.[1] Combining these two factors with the propellant mass fraction of the thrust arc (p_{mf}) the total propellant mass fraction (t_{mf}) becomes

$$t_{mf} = 1 - (1 - p_{mf}) \exp\left(\frac{-\Delta V_1}{g I_{sp1}}\right) \exp\left(\frac{-\Delta V_2}{g I_{sp2}}\right) \tag{2}$$

where ΔV_1 is equal to the launch V_∞ and ΔV_2 is equal to the arrival V_∞. For flyby missions, we drop the second exponential term.

We desire the lowest t_{mf} possible because it is the total mass fraction associated with propellant costs. The time of flight, though important, is not included in this cost function, so we evaluate a trajectory's total propellant mass fraction in conjunction with its time of flight. We select the most promising trajectories to use as initial guesses for the optimizer in step two.

Step 2: Optimization

We optimize the candidate LTGA trajectories found in step one using a direct method developed by Sims and Flanagan.[18] Their approach produced results comparable to SEPTOP[17] (Solar Electric Propulsion Trajectory Optimization Program), a well-tested optimization tool that uses the calculus of variations.

In the Sims-Flanagan LTGA trajectory model, each leg of the trajectory is subdivided into segments of equal duration. (A leg connects two mission bodies, such as an Earth-Mars leg.) The thrusting on each segment is modeled by an impulse at the midpoint of the segment, with conic arcs between the impulses. (In this aspect, the Sims-Flanagan model is similar to that used by Kawaguchi, Takiura, and Matsuo.[27]) In order to ensure that the spacecraft leaves the initial body of the leg and arrives at the final body of the leg, the first half of the leg is propagated forward from the initial body and the last half of the leg is propagated backward from the final body. In order for the trajectory to be feasible, the forward- and backward-propagated half-legs must meet at a "matchpoint" in the middle of the leg (see Fig. 1). This "matchpoint problem" can be stated as a constraint for the optimizer.

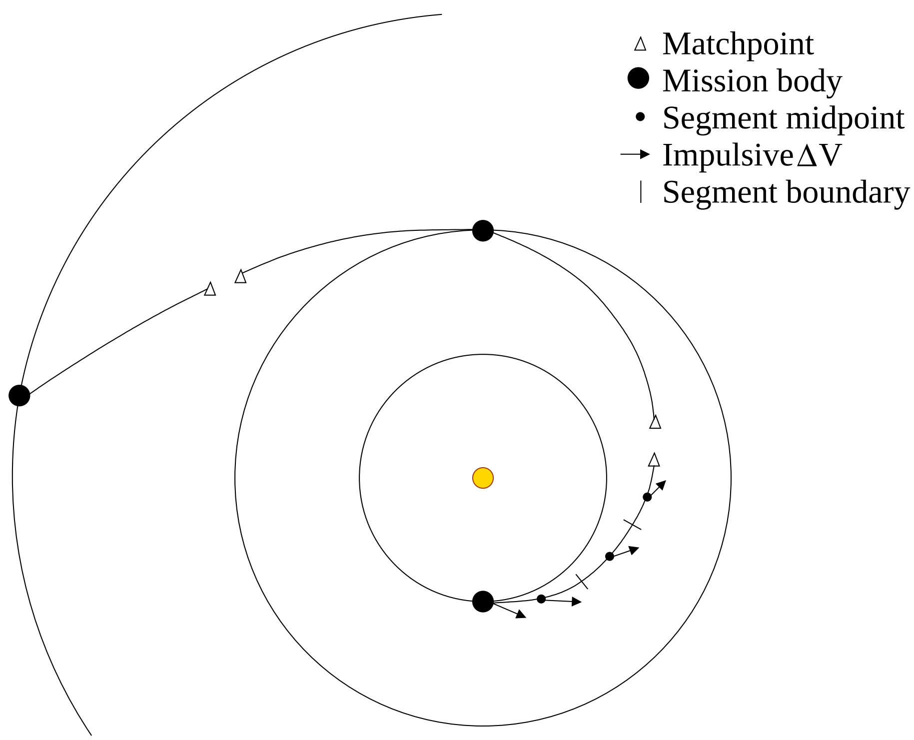


Figure 1: LTGA Trajectory Model (after Sims and Flanagan[18])

A gravity-assist maneuver is modeled as an instantaneous rotation of the $\mathbf{V}_\infty$. The magnitude and direction of the rotation are determined by the flyby periapsis altitude and the B-plane angle. The B-plane angle[28] is the angle between the $\mathbf{T}$ vector (a vector parallel to the ecliptic plane) and the $\mathbf{B}$ vector (the vector from the gravitating center to the target point).

The LTGA trajectory model uses the following variables:

- The impulsive $\Delta\mathbf{V}$ on each of the segments.
- The Julian dates at the launch, flyby, and destination bodies.
- The launch $\mathbf{V}_\infty$.
- The incoming inertial velocity vectors at all the post-launch bodies.
- The spacecraft mass at each body.
- The flyby periapsis altitude at the gravity-assist bodies.
- The B-plane angle at the gravity-assist bodies.

These variables can be altered by an optimizer to find the trajectory with the largest mass at the final body. Some constraints are imposed to make the resulting trajectory flyable. First, the position, velocity, and mass of the spacecraft must be continuous across the matchpoint in the middle of each leg (this is the "matchpoint problem" described above). Second, the ΔV magnitude on each segment must not exceed a certain value. This maximum value depends on the spacecraft mass and the available engine power, which in turn depends on the distance of the spacecraft from the sun. We can add other mission constraints very easily. For example, if we want a rendezvous with the destination body, a constraint that the arrival $\mathbf{V}_\infty = \mathbf{0}$ can be included.

The optimization software uses a solar array model that takes into account both the distance from the sun and the effect of temperature on solar cell efficiency. The same model is used by Williams and Coverstone-Carroll.[22] For most of the missions in this paper, we assume that the solar arrays produce 10 kW of power at 1 AU (the Earth-Mars-Ceres rendezvous mission being the only exception). Unless otherwise noted, we also assume that the housekeeping power required by the spacecraft is negligible.

We assume that each low-thrust engine needs at least 0.649 kW of power to operate and can use at most 2.6 kW (the values used by Sims and Flanagan in their original code). Figure 2 shows how the maximum power deliverable to the engine depends on the distance from the sun. The engine thrust and mass flow rate are modeled as linear functions of the power used, with the coefficients being the same as those used by Williams and Coverstone-Carroll.[22] The engine I_{sp} increases with the power used, so we assume that the maximum usable power is always used. When operated at 2.6 kW (which is often the case), the engine I_{sp} is 3300 s. A single engine is used for all missions considered in this paper.

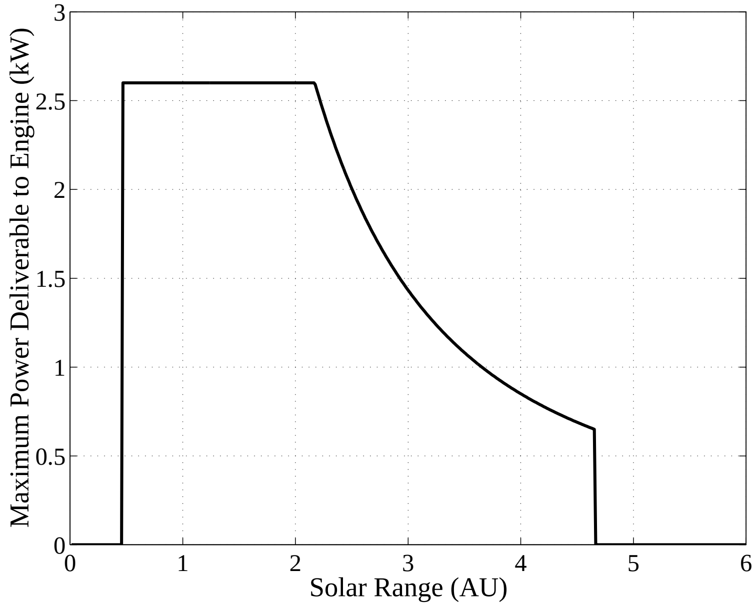


Figure 2: Power model (total power available at 1 AU is 10 kW)

We use the software NPOPT (Nonlinear Programming Optimizer) for the optimization. NPOPT is commercially available from Stanford Business Software Inc. It uses a sequential quadratic programming algorithm, which means it finds a locally optimal solution that is near the initial guess.

VERIFICATION OF THE OPTIMIZATION SOFTWARE

We call our optimization software GALLOP (Gravity-Assist Low-thrust Local Optimization Program). It is based heavily on the original Sims and Flanagan code. Nonetheless, it was entirely rewritten and thus required careful testing. The verification was accomplished by optimizing test cases and comparing the results to results from other software.

One of our test cases is a 2009 Earth-Mars-Vesta flyby mission that was investigated by Sims and Flanagan.[18] The launch V_∞ is fixed at 2.8 km/s. If we assume that the launch vehicle is a Delta 7326-9.5 (the vehicle that launched Deep Space I), then this launch V_∞ implies an initial spacecraft wet mass of 545 kg. The launch, flyby, and arrival dates are fixed at October 4, 2009, May 2, 2010, and January 27, 2011, respectively. We use 26 segments on the Earth-Mars leg and 38 segments on the Mars-Vesta leg (so that each segment has a duration close to 8 days).

The optimized final mass found by our software is 493.73 kg. The Sims and Flanagan software found precisely the same value (493.73 kg), whereas SEPTOP found a trajectory with a final mass of 493.74 kg.[18] A comparison of various trajectory characteristics is given in Table 1.

The good agreement found with the Earth-Mars-Vesta flyby test case is representative. In general, we found similar agreement between the optimal solutions found with our software and those found by other software for a number of different test cases (e.g. an Earth-Mars flyby, an Earth-Mars rendezvous, an Earth-Mars-Vesta rendezvous).

Table 1:
COMPARISON OF RESULTS FROM THREE OPTIMIZERS FOR AN EARTH-MARS-VESTA FLYBY MISSION

Optimization Software	Final mass (kg)	Mars flyby altitude[a] (km)	Launch $V_{\infty x}$ (km/s)	Launch $V_{\infty y}$ (km/s)	Launch $V_{\infty z}$ (km/s)
SEPTOP	493.74[18]	2818[29]	-0.810[29]	2.651[29]	0.397[29]
SDC[b]	494.05[30]	2815[30]	-0.808[29]	2.648[29]	0.420[29]
GALLOP	493.73	2838	-0.799	2.651	0.416

[a] Assuming a Mars radius of 3397 km.

[b] New optimization software currently being developed by G. Whiffen at JPL.[30]

NUMERICAL EXAMPLES

We now illustrate our approach to the design and optimization of LTGA trajectories with some examples.

A Rendezvous with Comet Tempel 1

Even though our tools were created for designing missions with multiple gravity-assists, we wanted to find out how well they work for designing direct missions. We decided to consider a direct mission which rendezvouses with Tempel 1. Designing such a mission is challenging due to the high inclination and eccentricity of Tempel 1 (currently 10.5° and 0.519, respectively).

We began by using STOUR to perform a broad search for trajectories between January 1, 2001 and January 3, 2016 (Julian dates 2451910 to 2457390). Figure 3 shows how the total propellant mass fraction (including launch, thrusting, and rendezvous propellant costs) depends on launch date. Each '×' in Fig. 3 corresponds to a different trajectory.

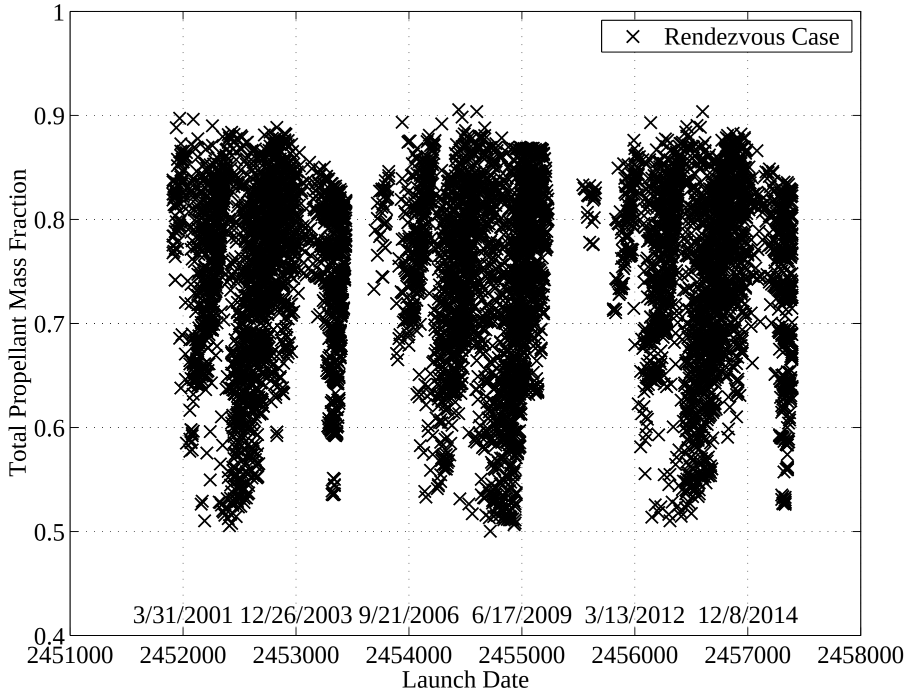
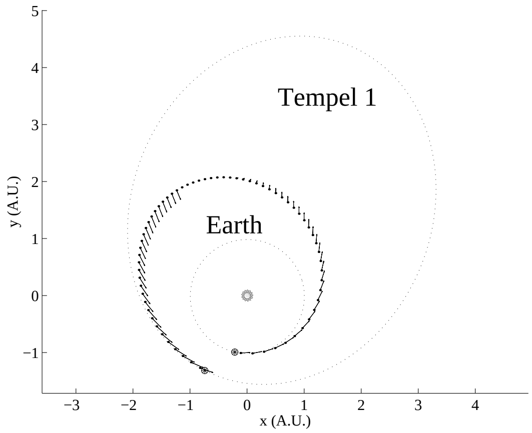


Figure 3: Low-Thrust Trajectories to Tempel 1 (Launch Dates between January 1, 2001 and January 3, 2016)

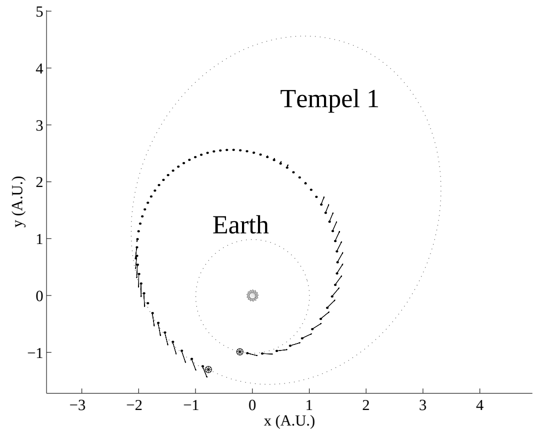
We see that there are three main groups of trajectories in the range of launch dates considered. For convenience, we will refer to them as group 1, group 2, and group 3, with group 1 having the earliest launch dates and group 3 the latest. This grouping occurs because Tempel 1 has an orbital period of about 5.5 years. Earth and Tempel 1 return to the same inertial positions at the same time every eleven years (after Tempel 1 completes two revolutions of the sun). Hence, every trajectory in group 3 corresponds to a trajectory in group 1 (eleven years earlier). This kind of correspondence does not exist between groups that are 5.5 years apart (e.g. between group 1 and group 2).

Nevertheless, we can find a trajectory in group 2 which has launch and arrival positions that are about the same as a trajectory in group 1. To ensure that the Earth is in the same position for the launch, the launch dates must be an integer number of years apart. To ensure that Tempel 1 is in the same position for the arrival, the arrival dates must be 5.5 years apart. This means that the time of flight (TOF) will differ between the trajectories from the two groups.

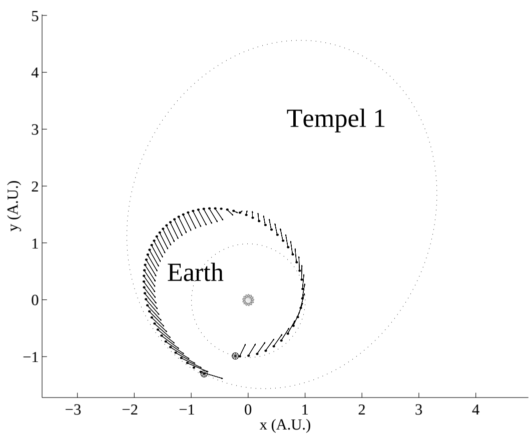
Figure 4 shows examples of four similar trajectories from the three groups. The trajectory from group 1 is selected as a representative case. The two trajectories from group 2 have launch dates that are five and six years after the launch date of the trajectory from group 1. Both group 2 trajectories have an arrival date that is 5.5 years after the arrival date of the group 1 trajectory. The group 3 trajectory occurs eleven years after the group 1 trajectory. The initial spacecraft masses are calculated from the launch V_∞'s assuming a Medlite launch vehicle.



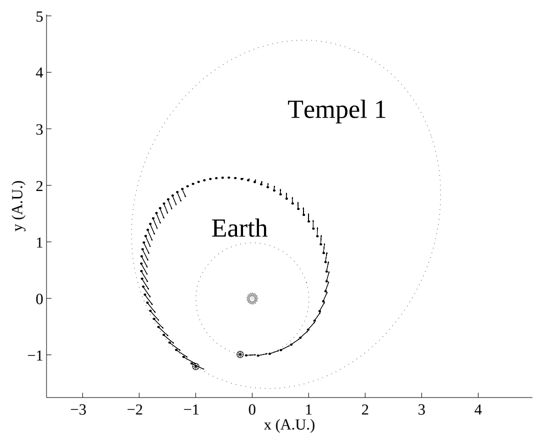
A Trajectory from Group 1.
Final mass = 331 kg. TOF = 2 years.
Launch V_∞ = 2.6 km/s.



Similar Trajectory from Group 2.
Final mass = 324 kg. TOF = 2.5 years.
Launch V_∞ = 3.2 km/s.



Similar Trajectory from Group 2.
Final mass = 197 kg. TOF = 1.5 years.
Launch V_∞ = 4.8 km/s.



Corresponding Group 3 Trajectory.
Final mass = 326 kg. TOF = 2 years.
Launch V_∞ = 2.8 km/s.

Figure 4: Examples of Similar Trajectories to Tempel 1

If we assume a specific launch vehicle (a Medlite), the optimizer can be used to study how the maximum mass deliverable to Tempel 1 depends on the launch date and arrival date. Figure 5 shows the results of such a study for trajectories in group 3 (launch dates in 2014).

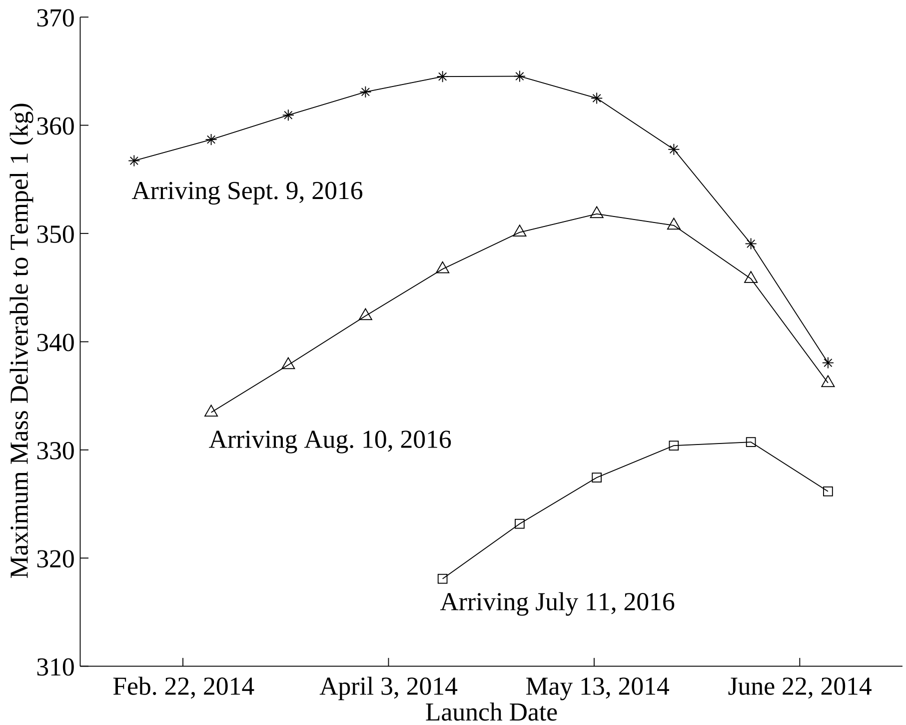


Figure 5: Maximum mass deliverable to Tempel 1 depending on launch and arrival dates

For each of the three arrival dates shown in Fig. 5, there is an optimal launch date. We find that if the arrival date is moved later, the optimal launch date moves earlier, so the optimal time of flight becomes longer. The optimal trajectory shown in Fig. 5 (launching April 28, 2014 and arriving Sept. 9, 2016) has a time of flight of 865 days. Optimal trajectories arriving later have a larger mass at Tempel 1, but they also have a longer time of flight.

A Rendezvous with Ceres via Mars

When the low-thrust version of STOUR was first created, it was tested by checking if it would find some of the optimized LTGA trajectories in the literature. One such trajectory, published by Sauer,[17] performs a gravity-assist maneuver at Mars and ends with a rendezvous at the asteroid Ceres. When STOUR does a broad search for Earth-Mars-Ceres rendezvous trajectories between 1999 and 2040,[13–15] the best launch date it finds is within days of the trajectory published by Sauer.

The most promising trajectory found by STOUR is used as an initial guess for our optimizer (GALLOP). The dates, launch mass, and launch V_∞ magnitude were all held fixed at the STOUR values. The spacecraft housekeeping power was assumed to be 125 W (the value used by Sauer). Table 2 summarizes the pertinent characteristics of the resulting optimized trajectory. We note that there is good agreement with the result found by Sauer. It should be emphasized that we did not allow the optimizer to vary the dates, the launch mass, or the launch V_∞ magnitude. Figure 6 shows a three-dimensional view of the spacecraft trajectory. The direction and relative magnitude of the impulsive ΔV's are indicated by line segments emanating from the segment midpoints.

Table 2:
EARTH-MARS-CERES RENDEZVOUS TRAJECTORY

	STOUR/GALLOP	Sauer[17]
Solar array power at 1 AU	5.11 kW[a]	5.00 kW
Launch date	May 6, 2003	May 2003
Launch V_∞	1.6 km/s	1.35 km/s
Initial mass	568 kg	568 kg
Mars flyby altitude	200 km[b]	N/A
Mars flyby V_∞	2.2 km/s	N/A
Time of flight (TOF)	2.9 years[c]	3.0 years
Final mass	412 kg	410 kg

[a] This was the lowest reference power for which a trajectory could be found (keeping the STOUR dates, launch mass, and launch V_∞ fixed).
[b] 200 km is the lower bound on the flyby altitude at Mars.
[c] The TOF found by STOUR was 3.1 years, but the optimized trajectory arrives at Ceres 0.2 years before the STOUR arrival date and then coasts.

A Low-Thrust Mission to Jupiter with Three Intermediate Flybys

One of the special capabilities of STOUR is its ability to find LTGA trajectories with multiple gravity-assist maneuvers (i.e. more than two). This capability is exemplified by the search for Earth-Venus-Earth-Mars-Jupiter (EVEMJ) flyby trajectories that was conducted by Petropoulos,[14] and Petropoulos and Longuski.[15]

The EVEMJ trajectory with the lowest in-plane propellant mass fraction is selected for optimization. As with the Earth-Mars-Ceres trajectory, the dates, launch mass, and launch V_∞ magnitude are all held fixed. The details of the resulting optimized trajectory are summarized in Table 3.

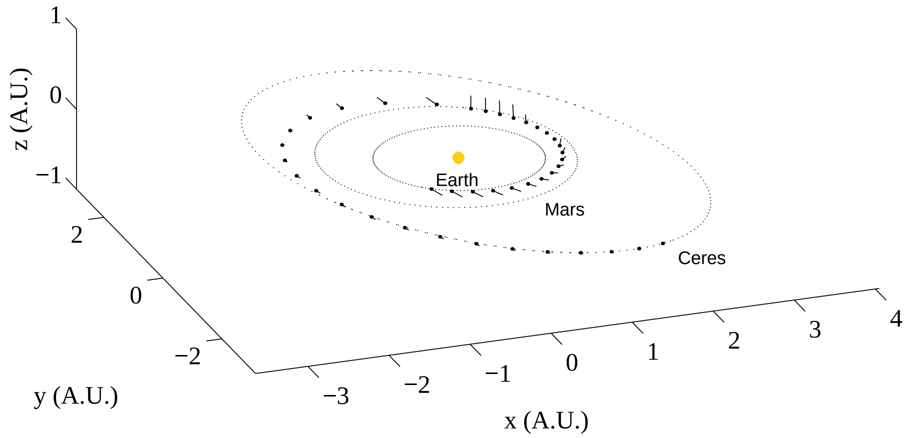


Figure 6: Earth-Mars-Ceres Rendezvous Trajectory

Figure 7 shows the optimized EVEMJ flyby trajectory. Figure 8 shows how the spacecraft specific energy varies during the course of the mission. The "Conic Arcs" are the conic sections connecting the impulsive ΔV's at the segment midpoints (see Fig. 1).

We note that there are three distinct thrusting phases on the optimized EVEMJ flyby trajectory. The first thrusting phase occurs between Venus and Earth and is used primarily to lower the energy. The second thrusting phase occurs near apoapsis between the Venus and Earth flybys. The third thrusting phase occurs just before and just after the Mars flyby. As shown in Fig. 8, the third thrusting phase increases the energy significantly.

Low-Thrust Trajectories to Pluto via Venus and Jupiter

We now consider low-thrust trajectories to Pluto using gravity-assist maneuvers at Venus and Jupiter. The results of a broad search for such trajectories are shown in Fig. 9. The search spans launch dates between January 1, 2001 and May 25, 2004 (Julian dates 2451910 to 2453150). Since there is no rendezvous at Pluto, the total propellant mass fraction (t_{mf}) includes only the launch and thrust-leg propellant costs. We note that the majority of trajectories found by STOUR occur in 2001.

There is a promising group of low-thrust EVJP trajectories occurring in late 2002 (low t_{mf} and low TOF). We choose one of these trajectories to optimize with GALLOP. We find that in order to create a feasible trajectory, the STOUR launch date has to be moved 41 days earlier. The launch V_∞ and all other dates are held fixed at the values found by STOUR. The characteristics of the resulting optimized trajectory

Table 3:
EVEMJ FLYBY TRAJECTORY

Solar Array Power at 1 AU	10 kW
Number of NSTAR Engines	1
Launch Date	Sept. 3, 2029
Launch V_∞	2.00 km/s
Initial Wet Spacecraft Mass	300.0 kg
Venus Flyby Date	Feb. 15, 2030
Venus Flyby V_∞	4.92 km/s
Venus Flyby Altitude	16861 km
Venus Flyby B-plane Angle[a]	-158.6°
Earth Flyby Date	Dec. 26, 2030
Earth Flyby V_∞	7.16 km/s
Earth Flyby Altitude	5678 km
Earth Flyby B-plane Angle[a]	-164.0°
Mars Flyby Date	May 26, 2031
Mars Flyby V_∞	12.65 km/s
Mars Flyby Altitude	200 km[b]
Mars Flyby B-plane Angle[a]	-6.4°
Jupiter Flyby Date	Jan. 20, 2035
Jupiter Flyby V_∞	6.03 km/s
Total Time of Flight	1965 days (5.4 years)
Final Dry Spacecraft Mass	228.8 kg
Propellant Mass Fraction	0.237

[a] Fundamental plane taken as ecliptic of J2000.

[b] 200 km is the lower bound on the flyby altitude at Mars.

are summarized in Table 4. The first two legs of the optimized trajectory are shown in Figure 10.

We note that on the optimized trajectory, the engine operates at maximum thrust from launch, through the Venus flyby, and until the spacecraft reaches about 3.6 AU. Then there is a short coasting phase. Finally, there is a short maximum-thrust phase until there is no longer enough power to run the engine (at 4.7 AU). The spacecraft coasts to the Jupiter gravity-assist maneuver and then coasts to Pluto.

It is difficult to compare this trajectory to other LTGA Pluto missions reported in the literature because they use more than one engine or more than one Venus flyby.[22] Our discovery of this Pluto trajectory is an encouraging result however, because it demonstrates the synergistic capabilities of STOUR and GALLOP.

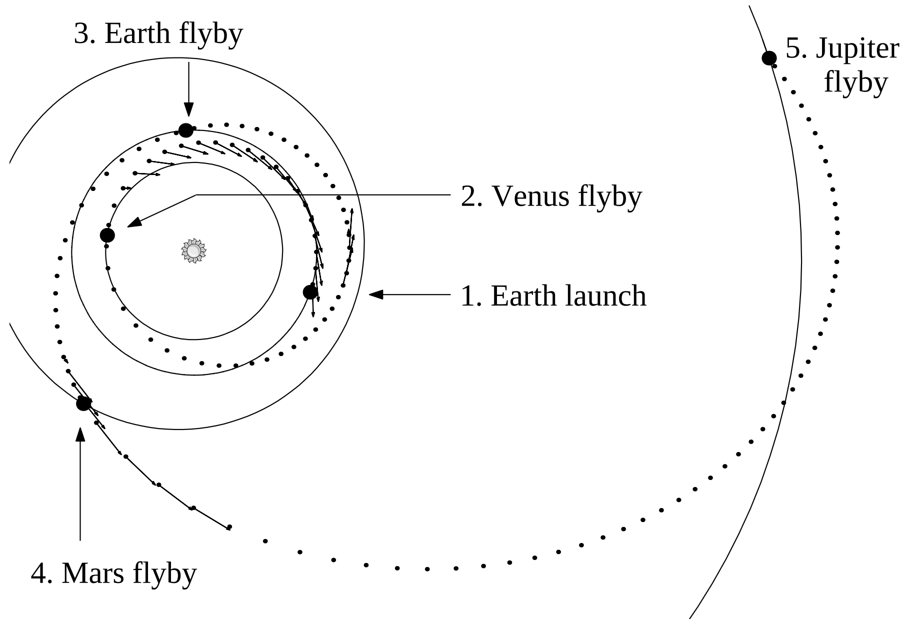


Figure 7: EVEMJ Flyby Trajectory

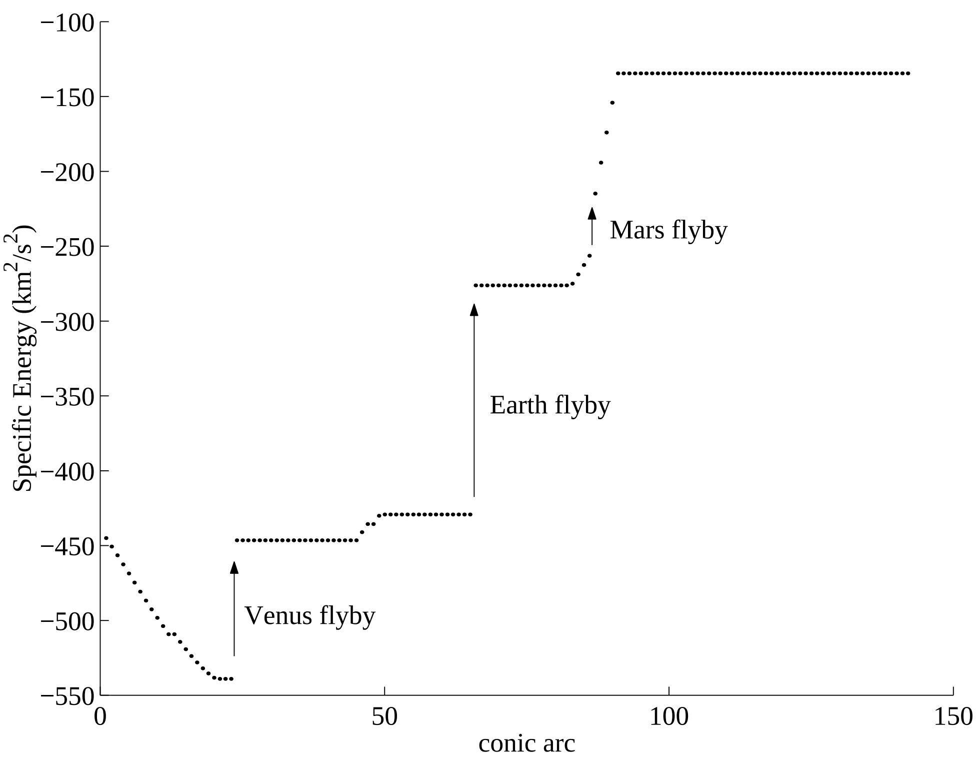


Figure 8: Energy Variation on the EVEMJ Flyby Trajectory

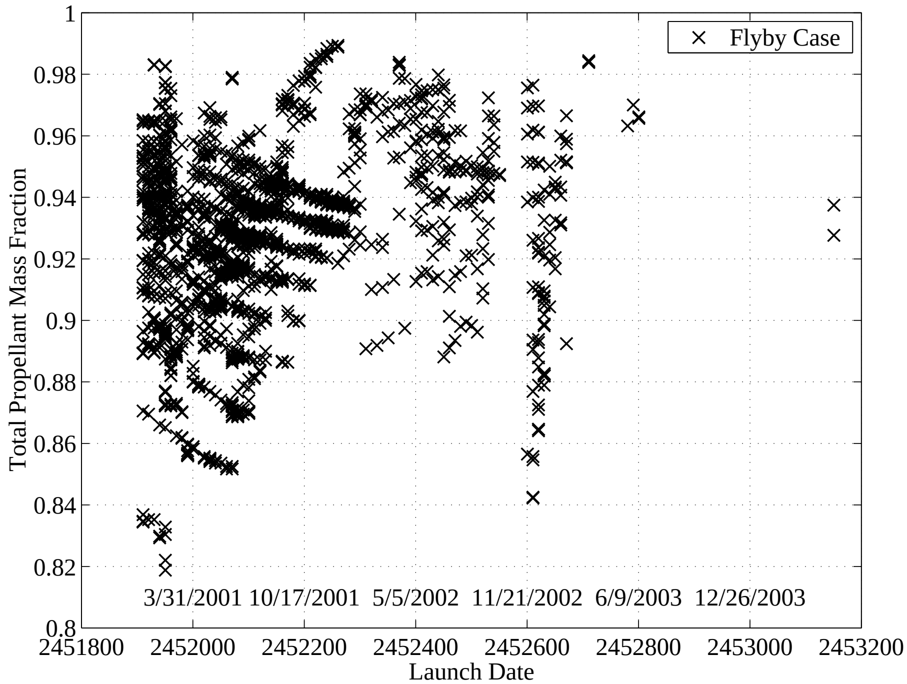


Figure 9: Earth-Venus-Jupiter-Pluto Trajectories (Launch Dates between January 1, 2001 and May 25, 2004)

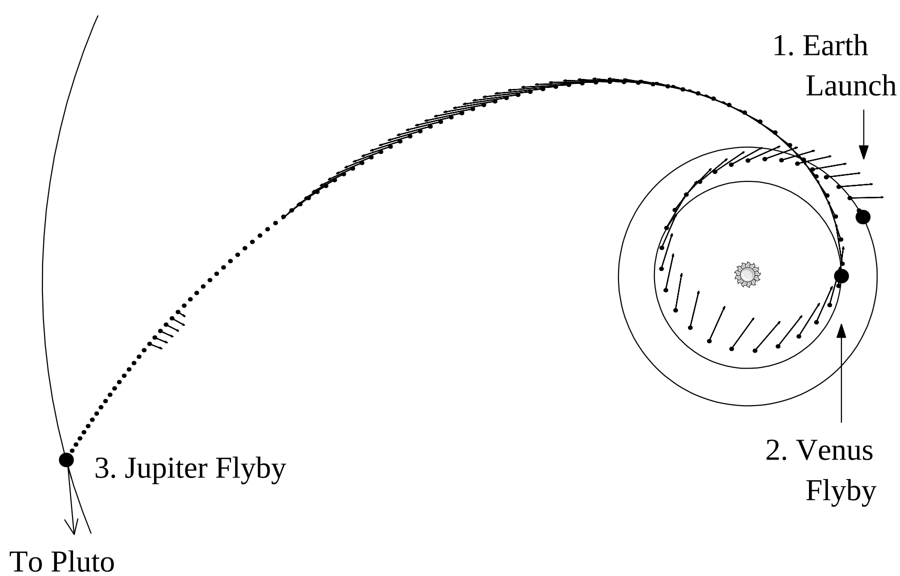


Figure 10: Earth-Venus-Jupiter-Pluto Trajectory

Table 4:
EVJP FLYBY TRAJECTORY

Solar Array Power at 1 AU	10 kW
Number of NSTAR Engines	1
Launch Date	Oct. 20, 2002
Launch V_∞	4.00 km/s
Initial Wet Spacecraft Mass	320.0 kg
Venus Flyby Date	May 19, 2003
Venus Flyby V_∞	8.18 km/s
Venus Flyby Altitude	200 km[a]
Venus Flyby B-plane Angle[b]	4.7°
Jupiter Flyby Date	April 21, 2005
Jupiter Flyby V_∞	10.00 km/s
Jupiter Flyby Altitude	548,450 km (7.7 R_J)
Jupiter Flyby B-plane Angle[b]	5.8°
Pluto Flyby Date	Nov. 28, 2014
Pluto Flyby V_∞	12.95 km/s
Total Time of Flight	4423 days (12.1 years)
Final Dry Spacecraft Mass	186.2 kg
Propellant Mass Fraction	0.418

[a] 200 km is the lower bound on the flyby altitude at Venus.

[b] Fundamental plane taken as ecliptic of J2000.

CONCLUSIONS

We are developing a new approach for designing low-thrust gravity-assist (LTGA) trajectories. It combines the broad search capabilities of STOUR with an efficient direct optimization program. We expect that this approach will have useful applications in the general problem of LTGA trajectories and will open new doors to the exploration of the solar system.

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