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ANALYSIS OF VARIOUS TWO SYNODIC PERIOD EARTH-MARS CYCLER TRAJECTORIES

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Trajectories that regularly encounter Earth and Mars but use small or no propulsive maneuvers are known as cycler trajectories, or cyclers. For cyclers that repeat after two Earth-Mars synodic periods, several variations are possible. A detailed investigation is presented of a simple two synodic period cycler, along with several promising variations using combinations of one-year and half-year phasing orbits. Analysis is included for both the circular coplanar model and a model with actual Earth and Mars ephemerides.

Introduction

A simple two Earth-Mars synodic period cycler is identified in an attempt to improve upon the characteristics of the Aldrin Cycler.[1,2] This cycler makes $3^2/_7$ revolutions about the Sun in the $4^2/_7$ years. Since the aphelion of this trajectory is slightly below the average distance of Mars from the Sun and the bending required at the Earth flyby is greater than the Earth can provide, some ΔV will be required. However, the V_∞ at both Earth and Mars is quite low compared to the Aldrin Cycler. In fact, the promise of this cycler is that the transfer legs between Earth and Mars are close to Hohmann transfers. Variations to this cycler which introduce additional Earth flybys with one-year and/or half-year phasing orbits are shown to improve the situation even more.

In order to encounter both the Earth and Mars at each transfer opportunity, all two synodic period cyclers require four vehicles in contrast to the Aldrin Cycler which requires only two. Two vehicles are on “Up” trajectories where the Earth-Mars transfer is Type I with short transfer times on the order of 9 months or less. Two other vehicles are on "Down" trajectories where the Mars-Earth transfers are Type I with short transfer times.

Circular Coplanar Methodology

In order to construct Earth-Mars cycler trajectories, we begin by making a number of simplifying assumptions:

1. The Earth-Mars synodic period is $2^1/_7$ years.
2. The orbits of Earth, Mars, and the cycler trajectory lie in the ecliptic plane.
3. Earth and Mars have circular orbits.
4. The cycler trajectory is conic and prograde (direct).
5. Only the Earth has sufficient mass to provide gravity-assist maneuvers.
6. Gravity-assist maneuvers occur instantaneously.

We note that assumption 1 is equivalent to assuming that the orbital period of Mars is $1^7/_8$ years (whereas a more accurate value is 1.881 years). This also leads to the inertial Earth-Mars geometry precisely repeating every 15 years.

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Simple Two Synodic Period Cycler (Case 1)

Figure 1 shows the simple two Earth-Mars synodic period cycler. In the circular coplanar model it has a period P=1.348 years, a radius of aphelion R_A=1.51 AU and the V_∞ at Earth is 5.6 km/s. For the Up transfer, the Earth-Mars transfer is Type I or II and the Mars-Earth leg is Type VI. The trajectory departs the Earth with the V_∞ inward of the Earth's velocity vector taking it through a perihelion of about 0.93 AU, crossing the Earth's orbit ahead of the Earth and outward to Mars' orbit. As seen from Figure 1 the transfer to Mars is about 225 degrees and takes a little over nine months. The trajectory continues onward making three complete orbits about the Sun without coming near either the Earth or Mars again until passing through its original starting point on the Earth's orbit for the third time, somewhat behind the Earth and finally encountering the Earth $^2/_7$ of a revolution about the Sun (102.9 deg.) from the starting point. The cycler has made 3 $^2/_7$ complete orbits about the Sun while Earth has made 4 $^2/_7$. The Earth flyby must now rotate the incoming V_∞ vector, which is outward, to the symmetrically inward orientation to begin the next cycle. Unfortunately, the rotation angle required is approximately 135 degrees and with a V_∞ of 5.65 km/s the Earth can only rotate the V_∞ vector about 82 degrees (assuming a minimum flyby altitude of 200 km).

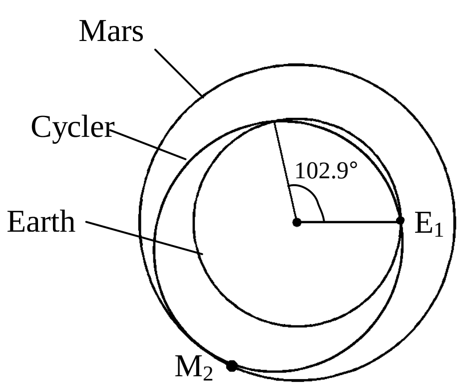

Figure 1: Simple two synodic period cycler (Case 1).

A more realistic model for the orbit of Mars is an ellipse with a semi-major axis of 1.524 AU, a perihelion of 1.381 AU and an aphelion of 1.666 AU. Thus the simple Case 1 cycler does not quite reach Mars' average distance from the Sun. It is clear that the elliptical-Martian-orbit version of the Case 1 cycler would require ΔV to make up for the inability of the Earth to rotate the V_∞ vector, as well as for the fact that over the course of seven cycles, of two synodic periods each, the Case 1 cycler will not make it to Mars' orbit more than one half of the time. In spite of these drawbacks, Case 1, as the simplest two synodic period cycler, provides a basis for more complicated variations.

Two Synodic Period Cycler With "Backflip" (Case 2)

Modifying Case 1 by introducing another Earth flyby, approximately six months and 180 degrees after the first, changes the situation somewhat. This six month, 180 degree transfer, or "backflip" trajectory, was first introduced for lunar trajectories by Uphoff.[3,4] The Up trajectory for this version leaves the Earth with a Type I or II short transfer to Mars and a Type V transfer back to Earth. This transfer from Earth to Mars to the first Earth encounter (at E_2 in Fig. 2) makes $2^{11}/_{14}$ revolutions about the Sun in $3^{11}/_{14}$ years. The Earth flyby then puts the vehicle onto a heliocentric orbit with a period of one year which re-encounters the Earth approximately six months and 180 degrees later, completing the $3^2/_7$ revolutions in $4^2/_7$ years. This second Earth flyby then sends the vehicle to the next Mars encounter, continuing the cycle. Fig. 2 shows this cycler trajectory. Note that the backflip trajectory is not shown since its difference from the Earth's orbit is primarily in the z-direction (out-of-plane.) The second Earth flyby and departure point for the second cycle is indicated by E_3. In the circular coplanar model the Earth-Mars-Earth trajectory has a period P=1.325 years, a radius of aphelion R_A=1.45 AU and the V_∞ at Earth is 4.15 km/s. For Case 2, the transfer does not reach Mars' orbit even in the circular coplanar model. In the elliptical-Martian-orbit model it does reach Mars but only when Mars is near its perihelion.

The lower V_∞ for Case 2 enables the Earth to rotate the V_∞ vector as much as about 102 degrees, thus easily enabling the first Earth flyby to rotate the incoming V_∞ to the required near polar orientation required for the backflip trajectory outgoing V_∞. The second Earth flyby is able to rotate the near polar incoming V_∞ to the outgoing V_∞ required for the transfer to the next Mars. Unfortunately, although Case 2 has many desirable characteristics, it cannot be used for an entire seven cycles. If fact it will reach Mars for at most two of the seven cycles without propulsive ΔV to augment the gravity assists.

R_A=1.65 AU and the V_∞ at Earth is 5.4 km/s. In this case the transfer reaches an aphelion approximately equal to Mars' aphelion and will thus always cross Mars orbit in the real world. Analysis of Case 3 with the accurate ephemerides of Earth and Mars is considered in more detail below.

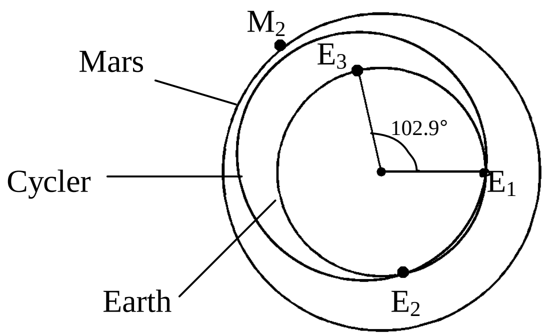

Figure 2: Two synodic period cycler with "backflip" (Case 2).

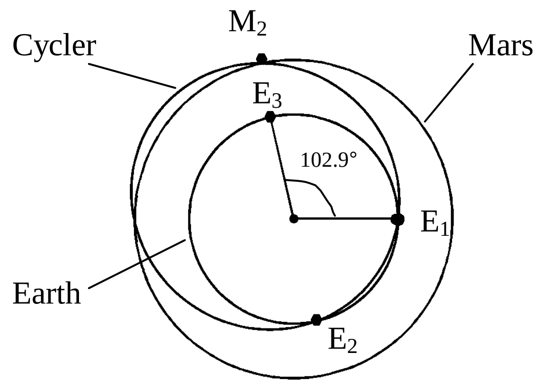

Figure 3: Two synodic period cycler with "backflip" plus 1-year loop (Case 3).

Two Synodic Period Cycler With "Backflip" Plus 1-Year Loop (Case 3)

Modifying Case 2 to introduce a third Earth flyby in addition to the "backflip" adds additional flexibility. This is accomplished by adding a one-year Earth-Earth loop either before or after the backflip. The order of the one-year loop and the backflip can be chosen to best advantage in the real world. The Up trajectory for this version leaves the Earth with a Type I short transfer to Mars and a Type III or IV transfer back to Earth. This transfer from Earth to Mars to the first Earth encounter (at E_2 in Fig. 3) makes $1^{11}/_{14}$ revolutions about the Sun in $2^{11}/_{14}$ years. The Earth flyby then puts the vehicle on a heliocentric orbit with a period of one year which re-encounters the Earth approximately six months and 180 degrees later and then re-encounters the Earth one year later, or vice versa. The final Earth flyby then sends the vehicle to the next Mars encounter. Figure 3 shows this cycler trajectory. Again as in Case 2, the backflip trajectory is not shown. The one-year Earth-Earth loop is also not shown since many different trajectories are possible. The final Earth flyby and departure point for the second cycle is indicated by E_3. In the circular coplanar model the Earth-Mars-Earth trajectory has a period P=1.484 years, a radius of aphelion

Two Synodic Period Cycler With One Or Two 1-Year Lops

Modifying Case 1 to introduce one or two one-year Earth-Earth loops or even a two-year Earth-Earth loop without a backflip is also possible. This leads however, to much higher V_∞ and less desirable characteristics than for Cases 1, 2 or 3, or the Aldrin Cycler for that matter.

Detailed Analysis Of Case 3

A detailed analysis of Case 3 was performed using the accurate ephemerides of the Earth and Mars. The trajectories were modeled as Sun-centered point-to-point conics connecting the Earth and Mars flybys. The flybys were modeled as instantaneous V_∞ rotations. This "V_∞-matching" model gives excellent insight into both the heliocentric and planetocentric trajectories and sufficient accuracy for developing long term trajectory scenarios that can be closely reproduced with fully numerically integrated trajectory models.

Table 1 shows data for a full cycle of seven two-synodic period cyclers (30 years). This should repeat every 30 years, with similar characteristics, since the Earth and Mars are very nearly at the same inertial positions every 15 years.

Table 1: Case 3 Cycler

Planet	Date ddmmyy	V_∞ km/s	Altitude km
Earth	8/25/05 22h	4.362	
Mars	2/21/06 3h	3.238	8000
Earth	6/5/08 19h	7.234	
Earth	12/5/09 16h	7.234	
Mars	6/2/10 12h	4.534	19000
Earth	8/19/12 10h	6.945	
Earth	2/18/14 7h	6.945	
Mars	7/1/14 8h	7.361	8000
Earth	11/30/16 15h	4.622	
Earth	6/1/18 12h	4.622	
Mars	9/15/18 10h	6.675	7000
Earth	3/30/21 23h	5.121	
Earth	9/29/22 20h	5.121	
Mars	4/21/23 16h	2.973	10000
Earth	6/28/25 17h	7.436	
Earth	12/28/26 14h	7.436	
Mars	6/10/27 21h	5.494	16000
Earth	9/15/29 1h	6.387	
Earth	3/16/31 22h	6.387	
Mars	7/15/31 2h	7.827	7000
Earth	1/8/34 23h	4.151	
Earth	7/10/35 20h	4.151	
Mars	11/13/35 17h	4.897	9000
Earth	5/3/38 0h	6.12	

It should be noted that the Earth-Earth transfers are constrained to be exactly 1.5 years apart. The choice of one-year loop or backflip and whether the backflip is "north" or "south" needs to be made in each case to make best use of the arrival and departure V_∞ to minimize the required bending by the Earth and potential required ΔV. The Mars flybys (given to the nearest 1000 km) are all at reasonably high altitudes. Whereas in the circular coplanar analysis the Mars flybys are arbitrarily high, in the real world the Mars gravity assist must control the inclination of the heliocentric orbit as well as adjust the energy slightly to properly phase for the next encounter. The Mars V_∞ vary between about 3 km/s and 8 km/s which compares to the value of 5.3 km/s in the circular coplanar case. The Earth V_∞ vary between about 4 km/s and 7.5 km/s which compares to 5.4 km/s. So while the real world solution values oscillate, they are on average similar to the prediction of the simpler model.

Ongoing And Future Work

Some initial optimal trajectory simulation of the Case 3 trajectories has begun. Initial results indicate that adjustments to the dates given in Table 1 will be necessary to minimize required deterministic ΔV. It may be possible, although it has not yet been demonstrated, that a completely ballistic trajectory may be attainable. Although the trajectory seems straightforward from the circular coplanar analysis, the interaction of the inclination and eccentricity of Mars' orbit make the actual trajectory quite complex. As can be seen from Fig. 3, the transfer from Mars to Earth is very nearly 180 degrees. Thus in the real world, a Type III or IV transfer must be chosen. This leads to multiple, distinct possible solutions, each with different characteristics.

The work of McConaghy, et al.,[5] identifies a cycler denoted the "ballistic S1L1 cycler." Figure 8 in that paper shows remarkable similarity to Case 3 in Fig. 3. In fact, they share significant similarities and one important difference. The Earth-Mars-Earth legs are nearly identical, however the ballistic S1L1 cycler has only two Earth flybys instead of three and the Earth-Earth transfer is not a resonance. That is, the transfer time and angle between the two Earth flybys is not an exact multiple or half-multiple of the Earth's period. This gives significantly more flexibility in the flyby dates of both Earth and Mars. A preliminary study underway has already identified a completely ballistic version of the ballistic S1L1 cycler over a 30-year period. These results and additional versions, both Up and Down of the cycler will be presented in a future paper.

It may turn out that the ultimate cycler uses judicious combinations of the various cyclers identified to take maximum advantage of the preferred characteristics of each. While such a trajectory is not strictly speaking a cycler, since it does not repeat exactly, it is a repeating trajectory in the sense of visiting Mars and Earth regularly on a two synodic period schedule.

Summary

Three versions of cycler trajectories taking two Earth-Mars synodic periods to repeat are analyzed. Detailed analysis in the circular coplanar model gives guidance as to the basic physical characteristics of the trajectories. Analysis with a realistic model of the Solar System including accurate ephemerides of the Earth and Mars and using point-to-point conic (V_∞-matching) modeling for the vehicle trajectory gives a very good representation of the actual trajectories.

The overall optimal real world cycler, over the full 30-year period necessary for a two synodic period cycler, may be a clever combination of the various possible cyclers.

Acknowledgments

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References

1. Aldrin, B., "Cyclic Trajectory Concepts," SAIC presentation to the Interplanetary Rapid Transit Study Meeting, Jet Propulsion Laboratory, Oct. 28, 1985.
2. Byrnes, D. V., Longuski, J. M., and Aldrin, B., "Cycler Orbit Between Earth and Mars," *Journal of Spacecraft and Rockets*, Vol. 30, No. 3, May-June 1993, pp. 334-336.
3. Uphoff, C. W., "The Art and Science of Lunar Gravity Assist," AAS Paper 89-170, AAS/GSFC International Symposium on Orbital Mechanics and Mission Design, Greenbelt, Md. April, 1989.
4. Uphoff, C. W. and Crouch, M.A., "Lunar Cycler Orbits with Alternating Semi-Monthly Transfer Windows", *Journal of the Astronautical Sciences*, 41, No. 2, Apr-June, 1993, pp. 189-205.
5. McConaghy, T. T., Longuski, J. M., and Byrnes, D. V., "Analysis of a Broad Class of Earth-Mars Cycler Trajectories," AIAA Paper 2002-4420, AIAA/AAS Astrodynamics Specialist Conference, Monterey, CA, Aug. 2002.