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DESIGN AND OPTIMIZATION OF LOW-THRUST GRAVITY-ASSIST TRAJECTORIES TO SELECTED PLANETS

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Highly efficient low-thrust engines are providing new opportunities in mission design. Applying gravity assists to low-thrust trajectories can shorten mission durations and reduce propellant costs from conventional methods. In this paper, an efficient approach is applied to the design and optimization of low-thrust gravity-assist trajectories to such challenging targets as Mercury, Jupiter, and Pluto. Our results for the missions to Mercury and Pluto compare favorably with similar trajectories in the literature, while the mission to Jupiter yields a new option for solar system exploration.

Introduction

THE design of low-thrust gravity-assist trajectories has proven to be a formidable task. Many researchers have responded to the challenge, but in spite of these efforts a perusal of the literature reveals a paucity of known results.[1-16]

Recently, new software tools have become available for the design and optimization of low-thrust gravity-assist (LTGA) trajectories. Petropoulos et al.,[9] Petropoulos and Longuski,[10,12] and Petropoulos[11] have developed a design tool which patches together low-thrust arcs, based on a prescribed trajectory shape. This method allows highly efficient, broad searches over wide ranges of launch windows and launch energies. A new, complementary tool, developed by Sims and Flanagan[14] and McConaghy et al.,[7] approaches the trajectory optimization problem by approximating low-thrust arcs as a series of impulsive maneuvers.

In this paper, we apply these tools to mission design studies of LTGA trajectories to Mercury, Pluto, and Jupiter (see Fig. 1). We demonstrate with these examples that our method provides an efficient way of designing and optimizing such trajectories with accuracy comparable to Refs. 13 and 15.

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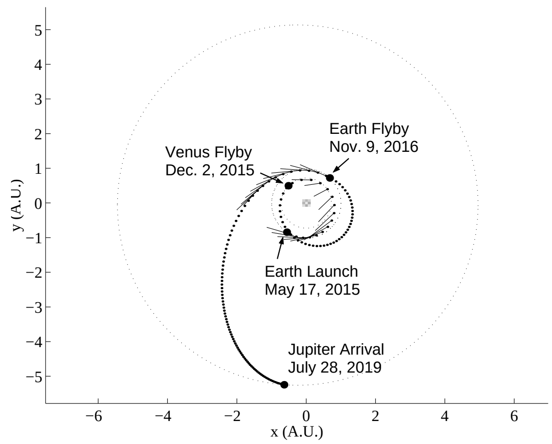

Fig. 1 Earth-Venus-Earth-Jupiter trajectory.

Methodology

The first step in designing an LTGA trajectory to a given target is to choose a sequence of gravity-assist bodies. While this problem has been examined for conic trajectories,[17] no prior work proposes a method for *path finding* in the case of LTGA missions. In our examples, we draw from two different resources for selecting paths. For the Mercury and Pluto missions, we take LTGA trajectories from the literature and attempt to reproduce those results. The path to Jupiter, however, is devised from studies of conic trajectories to the gas giant.[18]

Once a path has been chosen, we perform the next step: *path solving*. Employing a shape-based approach described below, we perform a broad search over the design space to discover sub-optimal LTGA trajectories. Evaluating the merits of these trajectories allows us to select a candidate for the third step: optimization. We optimize our trajectories to maximize the final spacecraft mass, which in turn may increase the scientific value of a mission.

Broad Search

We desire a computationally quick and efficient method of searching a broad range of launch dates and energies. For this process, we use a two-body patched-arc model. These arcs can be either coast only, thrust only, or a combination of the two. The coast arcs are standard Keplerian conic sections. The thrust arcs, however, are represented by *exponential sinusoids*[9] which are geometric curves parameterized in polar coordinates (r, θ) as

$$r = k_0 \exp[k_1 \sin(k_2\theta + \phi)] \quad (1)$$

where k_0, k_1, k_2, and ϕ are constants defining the shape and, consequently, the acceleration levels of the arc. These exponential sinusoids can be propagated analytically, which eliminates the need for time-consuming numerical propagation. The mathematical and algorithmic details are worked out in Refs. 9-12, and the algorithms are implemented as an extension of the Satellite Tour Design Program (STOUR).[19]

To evaluate the many candidate trajectories found by STOUR, we employ a cost function that computes the total propellant mass fraction due to the launch energy, thrust-arc propellant, and arrival V_∞ (if a rendezvous is desired).[7] Tsiolkovksy's rocket equation is used to account for the departure and arrival energies.[20,21] For the launch V_∞ (ΔV_1 in Eq. 2), we use a specific impulse of 350 seconds (I_{sp1}) to represent a chemical launch vehicle. A low-thrust specific impulse (I_{sp2}) of 3000 seconds is applied to the arrival V_∞ (ΔV_2) because we assume that the low-thrust arcs will remove any excess velocity in rendezvous missions. Finally, the thrust-arc propellant mass fraction given by STOUR (p_{mf}) yields the total propellant mass fraction (t_{mf}):

$$t_{mf} = 1-(1-p_{mf})\exp\left(\frac{-\Delta V_1}{gI_{sp1}}\right)\exp\left(\frac{-\Delta V_2}{gI_{sp2}}\right) \quad (2)$$

For flyby missions, ΔV_2=0. While Eq. 2 is only an approximation of the true propellant costs, it serves quite adequately to reduce the candidate trajectories to a manageable number among the myriad (up to tens of thousands) of possibilities produced by STOUR.

To reduce mission costs, the minimum total propellant mass fraction is desired. Since time of flight is not included in the cost function, it must be evaluated in conjunction with the total propellant mass fraction. This, of course, requires engineering judgment and a balancing of factors for a particular mission. It is up to the mission designer to judiciously select the most promising trajectories to use as initial guesses for optimization.

Optimization

Once good candidate trajectories are found, we optimize them with the direct method developed by Sims and Flanagan.[14] Our software, GALLOP (Gravity-Assist Low-thrust Local Optimization Program),[7] maximizes the final mass of the spacecraft. The essential features and assumptions of GALLOP are as follows (see Fig. 2).

- The trajectory is divided into legs between the bodies of the mission (e.g. an Earth-Mars leg).
- Each leg is subdivided into many short equal-duration segments (e.g. eight-day segments).
- The thrusting on each segment is modeled by an impulsive $\Delta\mathbf{V}$ at the midpoint of the segment. The spacecraft coasts on a conic arc between the $\Delta\mathbf{V}$ impulses.
- The first part of each leg is propagated forward to a matchpoint, and the last part is propagated backward to the matchpoint.
- Gravity-assist maneuvers are modeled as instantaneous rotations of the $\mathbf{V}_\infty$.

The optimization variables include the launch $\mathbf{V}_\infty$, the $\Delta\mathbf{V}$ on the segments, the launch,

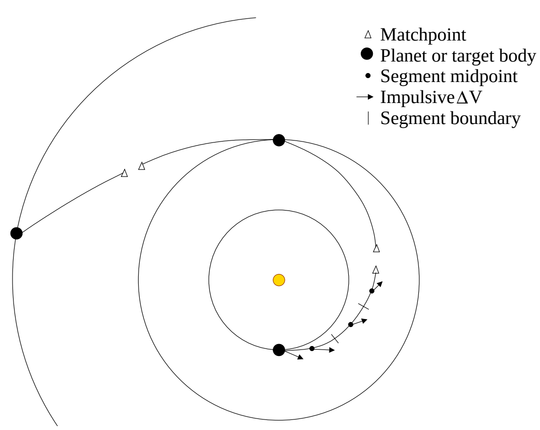

Fig. 2 LTGA trajectory model (after Sims and Flanagan[14]).

flyby and encounter dates, the flyby altitudes, the flyby B-plane angles, the spacecraft mass at each body, and the incoming velocity at each body. The initial spacecraft mass can also be determined with a launch vehicle model so that the injected mass is dependent upon the launch V_∞.

There are also two sets of constraint functions. One set ensures that the $\Delta\mathbf{V}$ impulses can be implemented with the available power. The other set enforces continuity of position, velocity, and spacecraft mass across the matchpoints (see Fig. 2). The optimizer uses a sequential quadratic programming algorithm to maximize the final mass of the spacecraft subject to these constraints.[7]

Reference 7 provides more details on the broad search and optimization procedures. Our method is also demonstrated on simple examples in Ref. 7. In this paper, we tackle more challenging problems using more mature versions of our software.

Optimized Trajectories

The success of our approach is illustrated in the following examples. The first two, missions to Mercury and to Pluto, are attempts to match or improve upon optimized results in the literature. The last example is a mission to Jupiter that we developed with no *a priori* knowledge of what the final, optimal result might be. In this case, we demonstrate the efficiency of our method by assessing the level of (computational and human) effort required to design and optimize the trajectory.

Earth-Venus-Mercury Rendezvous

Rendezvous missions to Mercury are particularly challenging for several reasons, including its proximity to the sun and the eccentricity and inclination of Mercury's orbit. Even assuming circular, coplanar orbits of both Earth and Mercury, a Hohmann transfer requires a launch V_∞ of at least 7.5 km/s, and the resulting arrival V_∞ magnitudes can be as high as 9.6 km/s.[22] The use of standard, chemical propulsion systems for removing this excess velocity on arrival can be prohibitive. Taking into account Mercury's highly eccentric and inclined orbit makes the rendezvous task even more daunting.

LTGA techniques offer low launch energies while Solar Electric Propulsion (SEP) systems can efficiently eliminate the arrival $\mathbf{V}_\infty$. Sauer incorporates a Venus flyby en route to Mercury to further facilitate the mission.[13] Sauer's optimized trajectory (using SEPTOP, an indirect method) has approximately 6.5 total revolutions around the sun, 5.75 of which occur on the Venus-Mercury leg of the mission. The "spiraling" in this trajectory is necessitated by the limited thrusting capability of a single SEP engine.[7,13,14]

We began our quest for an initial guess for GALLOP by performing a search in STOUR. We searched over a five-year period that included the launch date in Sauer's optimized trajectory.[13] Our Earth-Venus trajectory leg was a pure thrust leg, while the Venus-Mercury leg was a coast-thrust leg. We allowed the spacecraft to coast for more than one full revolution before starting to thrust at 0.68 AU. The coast time allows the exponential sinusoid geometry to be better aligned with the geometry of Mercury's orbit.[11] The additional coast revolution also provides GALLOP with more time to thrust.

Initially, we based our trajectory selection on the lowest total propellant mass fraction. Our attempts to optimize this case in GALLOP, however, were hindered by the thrust limitations of one SEP engine. Our trajectory had just under five total revolutions around the sun with only four complete revolutions on the Venus-Mercury leg. Given this trajectory profile, we concluded that the spacecraft could not provide sufficient acceleration to rendezvous with Mercury.

Reviewing our STOUR results, we sought a case with a higher number of Venus-Mercury revolutions. We limited our search to launch V_∞ magnitudes within 1 km/s of Sauer's value (of 2.3 km/s).[13] By taking the trajectory with the maximum number of Venus-Mercury revolutions in this launch V_∞ range, we obtained a case whose time of flight is only three days longer than Sauer's and whose launch date is less than four weeks earlier (see Table 1). In addition, this trajectory has 5.5 Venus-Mercury revolutions, which is only a quarter of a revolution less than Sauer's case.

With this initial guess, we were able to successfully optimize this trajectory in GALLOP. The initial launch mass is dependent upon the launch V_∞ magnitude via a Delta 7326 launch vehicle model. This is the Medlite Delta used by Sauer.[13] Sauer, however, uses a 10% launch margin so that the initial mass is only 90% of that available for a given launch V_∞. Our model uses the full 100% launch mass, so we expect our final results to be somewhat different than those of Ref. 13.

Figure 3 shows the final, optimized trajectory. Each dot represents the midpoint of the

Table 1 Earth-Venus-Mercury rendezvous trajectory

	STOUR	GALLOP	Sauer[13]
Earth launch date	Aug. 2, 2002	Aug. 13, 2002	Aug. 27, 2002
Launch V_∞	3.00 km/s	1.93 km/s	2.31 km/s
Launch vehicle	N/A[a]	Delta 7326	Delta 7326
Initial mass	N/A[a]	603 kg	521 kg
Earth-Venus time of flight (TOF)	184 days	198 days	185 days
Venus flyby date	Feb. 2, 2003	Feb. 27, 2003	Feb. 28, 2003
Venus flyby altitude	86,982 km	200 km[b]	NR[c]
Venus-Mercury TOF	667 days	655 days	663 days
Mercury arrival date	Nov. 30, 2004	Dec. 13, 2004	Dec. 22, 2004
Total TOF	851 days	853 days	848 days
Final mass	N/A[a]	416 kg	377 kg
Propellant mass fraction	0.526	0.310	0.276

[a] Not applicable.
[b] Flyby at altitude lower limit.
[c] Not reported.

segments described in the optimization section (see Fig. 2). The line emanating from a given dot shows the direction and relative magnitude of the thrust vector on that segment. In this case, each Earth-Venus segment represents 8.25 days while each Venus-Mercury segment is only 3.90 days. Our "rule of thumb" for selecting the number of segments on a given leg is to aim for an eight-day segment-length because this value has provided good results without necessitating an excessive number of variables.[7] (A large number of variables can slow GALLOP's computation time.) We decided, however, to use segments of about four days for the Venus-Mercury leg because of the high angular velocities close to the sun. Eight days is approximately 9% of a Mercurian year whereas the same amount of time is only about 2% of Earth's year. The shorter, four-day segments provide higher fidelity for the spiraling trajectory leg.

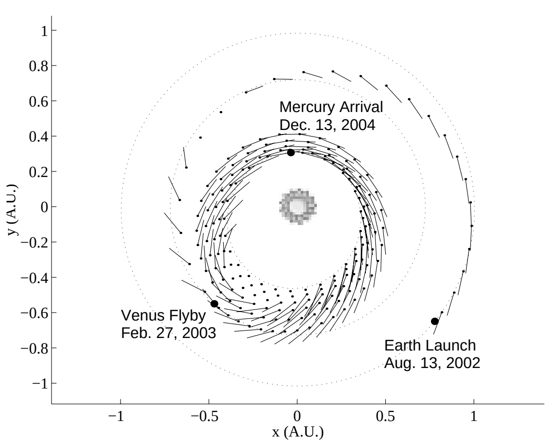

Fig. 3 Earth-Venus-Mercury trajectory.

Figure 3 also shows significant coast periods through each apoapsis of the Venus-Mercury leg. These coast periods are also evident in Sauer's trajectory.[13] This close correspondence in thrust profile suggests that both methods have found an optimal solution.

Table 1 shows good agreement between the results obtained from STOUR, GALLOP, and Sauer. We attribute STOUR's high propellant mass fraction to the trajectory shape imposed by the exponential sinusoid. It is expected that an optimized solution will improve upon this shape.

We also note that the final mass from GALLOP is actually higher than Sauer's, but this is offset by GALLOP's higher propellant mass fraction. While Sauer's trajectory has a higher launch V_∞ magnitude, GALLOP uses more thrust on the Earth-Venus leg. Sauer's optimal trajectory coasts for approximately the last third of the Earth-Venus leg, whereas the GALLOP trajectory has only a short coast phase on that leg (see Fig. 3). In a sense, GALLOP trades the additional launch energy for more SEP engine thrust time. These differences can account for GALLOP's earlier launch date. GALLOP needs the additional thrusting time to achieve the necessary change in energy for the Venus flyby (which occurs only 1 day earlier than the Venus flyby in Sauer's trajectory). We believe that many of the differences in the launch V_∞ magnitude, initial and final masses, propellant mass fraction, dates, and Earth-Venus thrust profile are due to the 10% launch vehicle margin in SEPTOP.

Earth-Venus-Jupiter-Pluto Flyby

Missions to Pluto present a different set of obstacles to mission designers, although the inclination and eccentricity of Pluto's orbit are again responsible for some of these difficulties. The chief obstacle, however, is Pluto's distance from the sun. Because the solar power available at large distances from the sun is not enough to operate the SEP engines, LTGA trajectories using SEP cannot provide a rendezvous with Pluto. Therefore, the trajectory we present in this paper has a Pluto flyby.

The search for a trajectory to Pluto began with a trajectory presented by Williams and Coverstone-Carroll.[15] Using the VARITOP optimization software, they optimized an LTGA mission to Pluto via gravity assists from Venus and Jupiter with a launch in June 2004.

We implemented this path (Earth-Venus-Jupiter-Pluto) in STOUR to find an initial guess for our optimizer. Unfortunately, STOUR could only produce results with long times of flight in the vicinity of the Williams and Coverstone-Carroll launch date because the exponential sinusoid is better at approximating trajectory arcs with low eccentricities. Trajectories to Pluto with reasonably short flight times, however, generally require high flyby velocities at Jupiter often resulting in heliocentric escape.[22] Although Ref. 22 did not address LTGA trajectories, the SEP engine's inability to thrust as far out as Jupiter means that a high V_∞ at Jupiter is required in our case, as well.

We accepted long flight times in STOUR with the understanding that the exponential sinusoid would not provide optimal energy increases on the Venus-Jupiter leg. In fact, the STOUR orbits from Jupiter to Pluto were actually elliptical. This accounts for the lengthy time of flight (TOF) of 6177 days on that leg in our chosen trajectory (see Table 2). STOUR managed to find trajectories with launch dates very close to those in Ref. 15, however. The minimum cost function trajectory in STOUR launches only two weeks prior to the trajectory in Ref. 15 and was selected as our initial guess for GALLOP.

With the STOUR initial guess, we were able to achieve an optimal solution in GALLOP using a Delta 7326 launch vehicle model to determine the initial mass (see Fig. 4). We note that any optimal result obtained in this case is dependent upon the bounds set on the Pluto arrival date. The final spacecraft mass increases as the arrival date is allowed to be pushed later by the optimizer. We accepted the 8.9 year TOF as a good, comparable result to the Williams and Coverstone-Carroll case (see Table 2).[15]

It should be noted that there are significant differences between our optimal result and that in Ref. 15. First of all, GALLOP uses a single

Table 2 Earth-Venus-Jupiter-Pluto flyby trajectory

	STOUR	GALLOP	VARITOP[15]
Earth launch date	May 25, 2004	May 17, 2004	June 8, 2004
Launch V_∞	7.5 km/s	5.28 km/s	5.73 km/s
Launch vehicle	N/A[a]	Delta 7326	Delta 7925
Initial mass	N/A[a]	331.9 kg	562.3 kg
Earth-Venus TOF	96 days	126 days	115 days
Venus flyby date	Aug. 28, 2004	Sept. 20, 2004	Oct. 1, 2004
Venus flyby altitude	5197 km	200 km[b]	300 km
Venus-Jupiter TOF	659 days	503 days	471 days[c]
Jupiter flyby date	June 18, 2006	Feb. 5, 2006	Jan. 2006
Jupiter flyby altitude	4.5 R_j[d]	7.2 R_j[d]	7.2 R_j[d]
Jupiter-Pluto TOF	6177 days	2614 days	2616 days[e]
Pluto arrival date	May 17, 2024	Apr. 3, 2013	Mar. 2013
Arrival V_∞	5.9 km/s	17.2 km/s	NR[f]
Total TOF	19 years	8.9 years	8.8 years
Final mass	N/A[a]	231.0 kg	420.2 kg
Propellant mass fraction	0.353	0.304	0.253

[a] Not applicable.
[b] Flyby at altitude lower limit.
[c] Estimated from Oct. 1, 2004 to Jan. 15, 2006. Specific Jupiter flyby date not reported in Ref. 15.
[d] Jovian radii. 1 R_j=71,492 km.
[e] Estimated from Jan. 15, 2006 to Mar. 15, 2013. Specific Jupiter and Pluto flyby dates not reported in Ref. 15.
[f] Not reported.

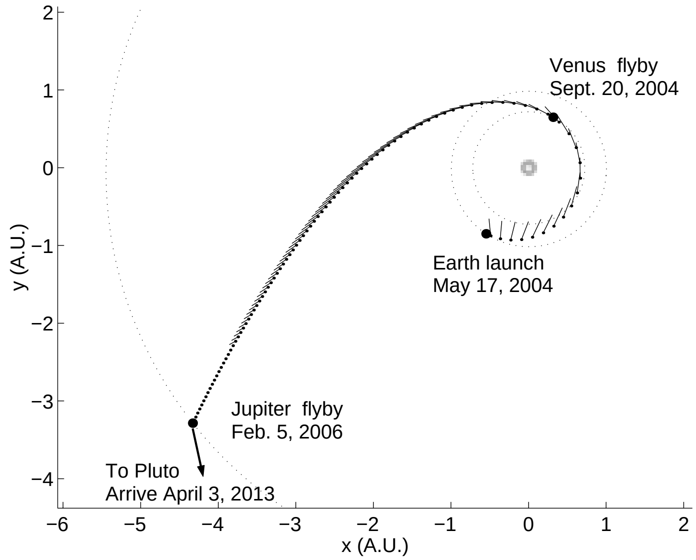

Fig. 4 Earth-Venus-Jupiter-Pluto trajectory.

SEP engine model. Second, we assumed the solar arrays could provide 10 kW of power at 1 AU. We also implemented the relatively small Delta 7326 launch vehicle model. A larger launch vehicle would have injected more mass for the same launch V_∞, but the single SEP engine would not have been able to provide sufficient thrust on such a large spacecraft.

Williams and Coverstone-Carroll, on the other hand, use three SEP engines and the more powerful Delta 7925 launch vehicle.[15] As with the trajectory to Mercury, this launch model includes a contingency margin – 14% in this case. The solar array power for their trajectory, though, was only 6 kW.[15] The lower solar array power means that the available thrust begins to decrease closer to the sun than with the 10 kW we assumed in GALLOP.

Because of the differences in launch vehicle, launch vehicle margin, number of SEP thrusters, and solar arrays it is difficult to draw any conclusions about the superiority of the either the GALLOP or VARITOP results. What our trajectory does show, however, is that it is possible to reach Pluto in less than nine years with a modest launch vehicle by employing LTGA techniques.

Earth-Venus-Earth-Jupiter Flyby

For our final trajectory we opted for a different approach to the path-finding problem. Instead of looking for LTGA paths already developed, we investigated paths previously developed for conic trajectories.[18] In particular, we looked for trajectories that required significant mid-course $\Delta\mathbf{V}$ maneuvers because we assumed that the SEP engine's capabilities would remove the need for such events.

Having selected Jupiter as our target body, we turned to work performed by Petropoulos et al.[18] for gravity-assist missions to Jupiter. In order to design and optimize a trajectory that could be flown before the end of the next decade, we looked for promising trajectories that launch in the years ranging from 2010 to 2020. Ref. 18 suggests that gravity assists from Venus and Earth can provide short flight times at the expense of a mid-course maneuver. We therefore chose to search for trajectories with the Earth-Venus-Earth-Jupiter (EVEJ) path in STOUR.

STOUR searched launch dates from January 1, 2009 to January 1, 2021 at a 10-day step. The search also included launch V_∞ values of 1.0 km/s to 4.5 km/s at a 0.5 km/s step. The arcs from Earth to Venus and Venus to Earth were pure thrust arcs. The Earth-Jupiter leg of the mission thrusted out to 5.0 AU where it switched to a coast arc. Because of the minimum power requirements of the SEP engine, it is unable to thrust past approximately 4.7 AU.[7] We allow STOUR to thrust a little longer to compensate for the sub-optimality of the exponential sinusoid shape.

Figure 5 shows the results of this STOUR search. This figure is a plot of the total flyby propellant mass fraction (t_{mf} in Eq. 2 with ΔV_2=0) versus launch date. Because over 16,000 trajectories resulted from this search, we use this cost

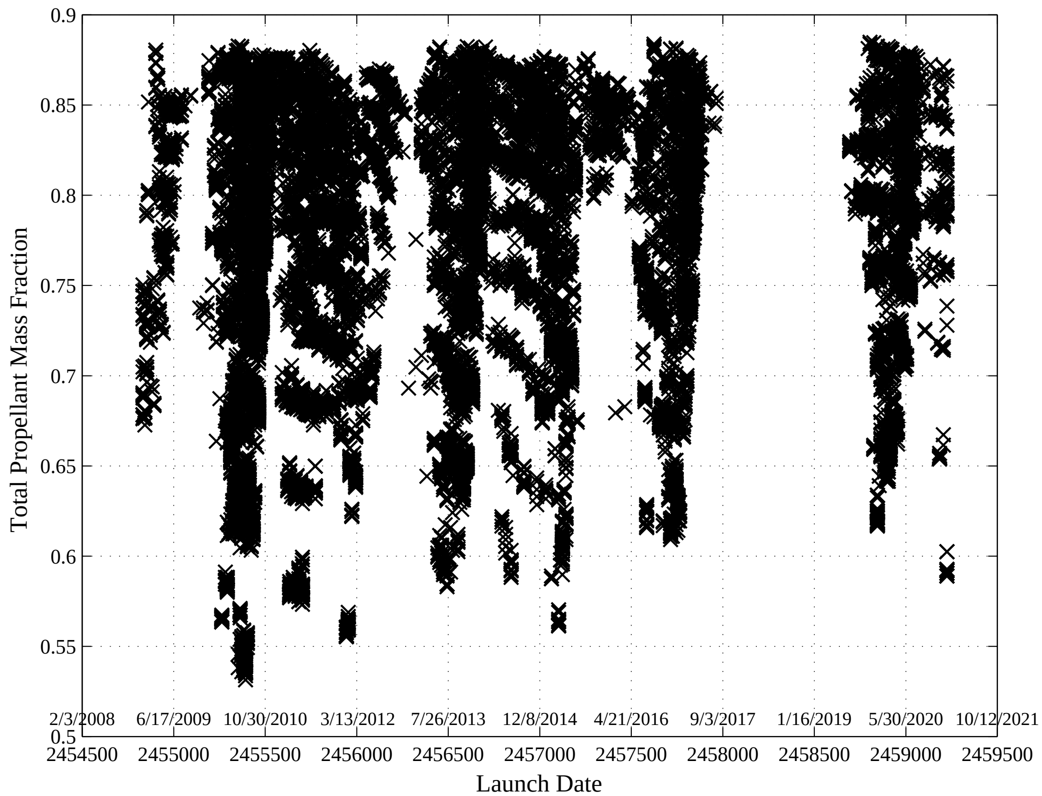

Fig. 5 Earth-Venus-Earth-Jupiter flyby cost function.

function to help determine which regions of the design space are the most promising.

Since we desire a trajectory with a low TOF, we do not simply select the trajectory with the lowest total propellant mass fraction. In addition, we consider the accelerations on each trajectory. STOUR computes the maximum and average acceleration values on each trajectory to indicate the amount of thrust needed to maintain the exponential sinusoid shape. Since we assume a single thruster in GALLOP, we generally opt for STOUR trajectories with low acceleration values. For the EVEJ case, the trajectories with low cost functions (in Fig. 5) that launch shortly after Dec. 8, 2014 stand out as having both short TOFs and low accelerations. Performing a more refined, one-day step search over launch dates from Jan. 1, 2015 to Aug. 1, 2015 resulted in the STOUR initial guess shown in Table 3.

Using this STOUR guess, GALLOP obtained an optimal solution launching only one week later than the STOUR trajectory. Figures 1 and 6 also show the thrust profile from the GALLOP result. This thrust profile is plotted in Fig. 7 as the ΔV magnitude on each segment (illustrated in Fig. 2). The maximum possible ΔV magnitude on a given segment is dependent on three main factors: 1) the amount of time represented by a given segment, 2) the mass of the spacecraft on that segment, and 3) the distance from the sun.

The first and third legs of the trajectory have higher ΔV magnitudes because more days are represented by these segments. The Earth-Venus and Earth-Jupiter legs have approximately 12-day segments, whereas the Venus-Earth leg has segments of about 8 days. The leg lengths are different for two reasons. The first is that longer segments were used on the Earth-Jupiter leg because of the lower velocities at this distance from the sun. Second, GALLOP has the freedom to change the flyby dates but not alter the number of segments on each leg. The Venus flyby date was moved so much later than the STOUR initial guess that the segments lengthened from just under eight days to over twelve days.

The maximum ΔV available increases throughout the first leg and the beginning of the third leg because the mass decreases as propellant is expelled (see Fig. 7). While the thrust is

Table 3 Earth-Venus-Earth-Jupiter flyby trajectory

	STOUR	GALLOP	Mystic[23]
Earth launch date	May 9, 2015	May 17, 2015	May 16, 2015
Launch V_∞	2.0 km/s	1.80 km/s	1.78 km/s
Launch vehicle	N/A[a]	Delta 7326	Delta 7326
Initial mass	N/A[a]	609.6 kg	610.9 kg
Earth-Venus TOF	119 days	199 days	200 days
Venus flyby date	Sept. 5, 2015	Dec. 2, 2015	Dec. 2, 2015
Venus flyby altitude	4481 km	200 km[b]	670 km
Venus-Earth TOF	345 days	343 days	342 days
Earth flyby date	Aug. 15, 2016	Nov. 9, 2016	Nov. 8, 2015
Earth flyby altitude	4219 km	300 km[b]	300 km[b]
Earth-Jupiter TOF	1027 days	991 days	966 days
Jupiter arrival date	June 8, 2019	July 28, 2019	July 2, 2019
Arrival V_∞	5.97 km/s	5.81 km/s	5.88 km/s
Total TOF	1491 days, 4.1 years	1533 days, 4.2 years	1508 days, 4.1 years
Final mass	N/A[a]	537.9 kg	538.7 kg
Propellant mass fraction	0.485	0.118	0.118

[a] Not applicable.
[b] Flyby at altitude lower limit.

constant over these periods, the lower spacecraft mass on successive segments means a greater ΔV is imparted by this thrust.

The third factor controlling the maximum available thrust is the distance from the sun. We model SEP engines that need a minimum amount of power to operate. (Solar array power is proportional to the inverse-square of the radius from the sun.) While there is sufficient power to operate at maximum thrust well past Mars' orbit, at about 2.2 AU the thrust levels drop off because of the loss of power, as can be seen in Fig. 7.

Figure 7 also shows that while the Venus-Earth STOUR leg was a pure thrust leg, GALLOP made that leg a pure coast arc. Furthermore, GALLOP cut off the thrust on the Earth-Jupiter leg at about 1.6 AU as opposed the 5.0 AU cut-off in STOUR. By moving the Venus and Earth flyby dates several months later and using this improved thrust profile, GALLOP reduces the propellant mass fraction from 0.485 to 0.118 (see Table 3).

The GALLOP results also compare well with a recent result obtained using Mystic, an optimization program (which propagates trajectories numerically) currently being developed by Whiffen at the Jet Propulsion Laboratory.[23] Table 3 shows that the Mystic launch date differs by only one day from the GALLOP launch date, and the final mass differs by less than 1 kg, or 0.1%. The most noticeable difference between the two trajectories is the Jupiter arrival date.

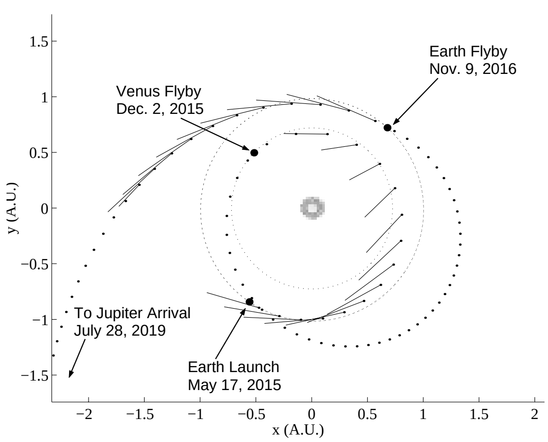

Fig. 6 Earth-Venus-Earth-Jupiter trajectory.

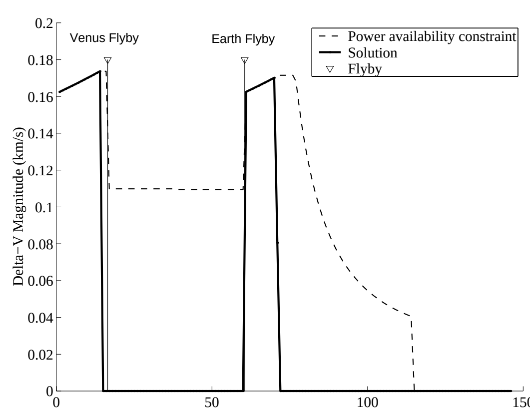

Fig. 7 Earth-Venus-Earth-Jupiter ΔV profile.

The Mystic trajectory arrives at Jupiter about 26 days sooner, but it has an additional 70 m/s in flyby V_∞. GALLOP can obtain a trajectory with a Jupiter flyby date of July 8, 2019 (only 6 days after Mystic's flyby date) if we are willing to sacrifice 0.2 kg of the final mass. This shorter, but slightly less optimal, trajectory would also cost about 50 m/s in the arrival V_∞ magnitude. The arrival V_∞ magnitude is important if Jupiter orbit insertion is required.

To give an idea of the efficiency of our method, which involves the interplay of human labor and computer time, we provide the following information. The initial STOUR run, that yielded the 16,000 trajectories shown in Fig. 5, took approximately 8 hours of run-time on a single 600 MHz processor of a Sun Blade 1000 (operating at about 50 Mflops/sec). The optimized GALLOP trajectory was obtained by one user in about four workdays.

Conclusions

The shape-based method proves to be a very efficient approach to finding good LTGA trajectory candidates for optimization. Our optimization software accepts these initial guesses and usually converges to an optimal solution in a reasonable period of time. While the challenge of LTGA trajectory design and optimization is still a difficult one, we believe we have made progress in facilitating the process. We hope our method proves beneficial in the planning of new and exciting deep-space missions.

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